

Problemas – Tema 5

Solución a problemas de Integrales - Hoja 08 - Problemas 3

Hoja 8. Problema 3

3. Calcula.

a) $\int \sqrt{e^x+1} dx$

b) $\int \frac{1}{\sqrt[4]{x}-3} dx$

c) $\int \frac{\sqrt{x}}{3x+1} dx$

a) $I = \int \sqrt{e^x+1} dx \rightarrow$ cambio de variable $e^x+1=t^2 \rightarrow e^x dx = 2t dt$

$$dx = \frac{2t dt}{e^x} \rightarrow dx = \frac{2t dt}{t^2-1}$$

$$I = \int \sqrt{t^2} \frac{2t dt}{t^2-1} \rightarrow I = \int \frac{2t^2}{t^2-1} dt \rightarrow I = 2 \int \frac{t^2}{t^2-1} dt \rightarrow I = 2 \int \frac{t^2-1+1}{t^2-1} dt$$

$$I = 2 \int dt + 2 \int \frac{1}{t^2-1} dt \rightarrow I = 2t + 2 \int \frac{1}{(t+1)(t-1)} dt$$

Aplicamos método de coeficientes indeterminados.

$$\frac{1}{(t+1)(t-1)} = \frac{A}{t+1} + \frac{B}{t-1} \rightarrow 1 = A(t-1) + B(t+1)$$

$$t=1 \rightarrow 1 = 0 + 2B \rightarrow B = \frac{1}{2}$$

$$t=-1 \rightarrow 1 = -2A \rightarrow A = -\frac{1}{2}$$

$$I = 2t + 2 \cdot \frac{-1}{2} \int \frac{1}{t+1} dt + 2 \cdot \frac{1}{2} \int \frac{1}{t-1} dt = 2t - \ln|t+1| + \ln|t-1| + C = 2t + \ln \left| \frac{t-1}{t+1} \right| + C$$

Deshacemos el cambio de variable $\rightarrow e^x+1=t^2 \rightarrow \sqrt{e^x+1}=t$

$$I=2\sqrt{e^x+1}+\ln\left|\frac{\sqrt{e^x+1}-1}{\sqrt{e^x-1}+1}\right|+C$$

b) $I=\int \frac{1}{\sqrt[4]{x-3}} dx \rightarrow$ cambio de variable $x=t^4 \rightarrow dx=4t^3 dt$

$$I=\int \frac{1}{\sqrt[4]{t^4-3}} 4t^3 dt \rightarrow I=\int \frac{4t^3}{t-3} dt=4\int \frac{t^3}{t-3} dt$$

Dividimos los polinomios $\rightarrow \frac{t^3}{t-3}=t^2+3t+9+\frac{27}{t-3}$

$$I=4\int \frac{t^3}{t-3} dt=4\int (t^2+3t+9)dt+4\cdot 27\int \frac{1}{t-3} dt=\frac{4}{3}t^3+6t^2+36t+108\ln|t-3|+C$$

Deshacemos el cambio de variable $\rightarrow x=t^4 \rightarrow \sqrt[4]{x}=t$

$$I=\frac{4}{3}x^{3/4}+6x^{1/2}+36x^{1/4}+108\ln|x^{1/4}-3|+C$$

c) $\int \frac{\sqrt{x}}{3x+1} dx \rightarrow$ cambio de variable $\rightarrow x=t^2 \rightarrow dx=2t dt$

$$I=\int \frac{\sqrt{t^2}}{3t^2+1} 2t dt=\int \frac{2t^2}{3t^2+1} dt=2\int \frac{t^2}{3t^2+1} dt=\frac{2}{3}\int \frac{3t^2}{3t^2+1} dt=\frac{2}{3}\int \frac{3t^2+1-1}{3t^2+1} dt$$

$$I=\frac{2}{3}\int dt-\frac{2}{3}\int \frac{1}{3t^2+1} dt=\frac{2}{3}t-\frac{2}{3}\int \frac{1}{1+(\sqrt{3}t)^2} dt=\frac{2}{3}t-\frac{2}{3\cdot\sqrt{3}}\int \frac{\sqrt{3}}{1+(\sqrt{3}t)^2} dt$$

$$I=\frac{2}{3}t-\frac{2\cdot\sqrt{3}}{9}\operatorname{arctg}(\sqrt{3}t)+C$$

Deshacemos el cambio de variable $\rightarrow x=t^2 \rightarrow \sqrt{x}=t$

$$I = \frac{2}{3}\sqrt{x} - \frac{2 \cdot \sqrt{3}}{9} \operatorname{arctg}(\sqrt{3x}) + C$$