

Problemas – Tema 5

Solución a problemas de Integrales - Hoja 23 - Problemas 5, 6, 7, 8

■ Hoja 23. Problema 5

5. $\int \frac{4x^3}{x^2+x} dx$

$$I = \int \frac{4x^3}{x^2+x} dx = 4 \int \frac{x^2}{x+1} dx \rightarrow \text{Realimos el cociente de polinomios}$$

$$\frac{x^2}{x+1} = x - 1 + \frac{1}{x+1} \rightarrow I = 4 \int \left(x - 1 + \frac{1}{x+1} \right) dx = 4 \left[\int x dx - \int dx + \int \frac{1}{x+1} dx \right]$$

$$I = 2x^2 - 4x + 4 \ln|x+1| + C$$

■ Hoja 23. Problema 6

6. $\int \operatorname{sen}(x) \cdot \ln(\cos(x)) dx$

Integramos por partes.

$$u = \ln(\cos(x)) \rightarrow u' = \frac{-\operatorname{sen}(x)}{\cos(x)}$$

$$v' = \operatorname{sen}(x) \rightarrow v = -\cos(x)$$

$$I = u(x) \cdot v(x) - \int v(x) \cdot u'(x) dx = -\cos(x) \cdot \ln(\cos(x)) - \int \cos(x) \cdot \frac{\operatorname{sen}(x)}{\cos(x)} dx$$

$$I = -\cos(x) \cdot \ln(\cos(x)) - \int \operatorname{sen}(x) dx$$

$$I = -\cos(x) \cdot \ln(\cos(x)) + \cos(x) + C = \cos(x)[1 - \ln(\cos(x))] + C$$

Hoja 23. Problema 7

7. $\int x \cdot \ln(\sqrt{1+x^2}) dx$

Integramos por partes.

$$u = \ln(\sqrt{1+x^2}) \rightarrow u' = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}$$

$$v' = x \rightarrow v = \frac{x^2}{2}$$

$$I = u(x) \cdot v(x) - \int v(x) \cdot u'(x) dx = \frac{x^2 \ln(\sqrt{1+x^2})}{2} - \int \frac{x^2}{2} \cdot \frac{x}{\sqrt{1+x^2}} dx = \frac{x^2 \ln(\sqrt{1+x^2})}{2} - \frac{1}{2} \int \frac{x^3}{\sqrt{1+x^2}} dx$$

Realizamos la división de polinomios.

$$\frac{x^3}{1+x^2} = x - \frac{x}{1+x^2}$$

$$I = \frac{x^2 \ln(\sqrt{1+x^2})}{2} - \frac{1}{2} \int x dx + \frac{1}{2} \int \frac{x}{1+x^2} dx = \frac{x^2 \ln(\sqrt{1+x^2})}{2} - \frac{1}{4} x^2 + \frac{1}{4} \ln|1+x^2| + C$$

Hoja 23. Problema 8

8. $\int x^2 e^{2x} dx$

Integramos por partes.

$$u = x^2 \rightarrow u' = 2x$$

$$v' = e^{2x} \rightarrow v = \frac{1}{2} e^{2x}$$

$$I = u(x) \cdot v(x) - \int v(x) \cdot u'(x) dx = \frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx$$

Volvemos a aplicar partes.

$$u = x \rightarrow u' = 1$$

$$v' = e^{2x} \rightarrow v = \frac{1}{2} e^{2x}$$

$$I = \frac{1}{2} x^2 e^{2x} - \left[\frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx \right] = \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C = \frac{1}{2} e^{2x} \left(x^2 - x + \frac{1}{2} \right) + C$$